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FIXED POINT THEOREMS OF SOME MAPPINGS VIA INTERPOLATION IN COMPLETE DISLOCATED b-METRIC SPACES

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ABSTRACT. In this paper, we introduce interpolative Kannan type contraction mappings on a complete dislocated b-metric space. In particular, we discuss the existence of a fixed point of such a mapping in a complete dislocated b-metric space. Moreover, we introduce interpolative Reich-Rus-Ćirić type contraction mappings and interpolative Hardy-Rogers type contraction mappings, Kannan-Ćirić type contraction mappings and interpolative Kannan-Meir-Keeler type contractions on a complete dislocated b-metric space. In particular, we prove the existence of a fixed point of such mappings in a complete dislocated b-metric space.

Keywords. Mappings, dislocated b-metric spaces, fixed point theorems.

1. INTRODUCTION

Hitzler [9] introduced the notion of dislocated metric spaces. Recently, Hitzler and Seda [8] established that a contraction mapping has a unique fixed point in complete dislocated metric spaces. Dislocated metric space plays a crucial role in topology, logical programming and electronics engineering. In [15], Zeyada, Hassan and Ahmed gave a generalization of a fixed point theorem due to Hitzler and Seda in complete dislocated quasi metric spaces. Recently, Hussain et al. [10] introduced the concept of dislocated b-metric spaces. They proved common fixed point results for weakly contractive mappings on ordered dislocated b-metric spaces.

In this paper, we prove some results on fixed point theorems for some mappings via interpolation in complete dislocated b-metric spaces.

We continue with some preliminaries:

Definition 1 ([10]). *Let $\mathcal{X}$ be a non-empty set and let $d : \mathcal{X} \times \mathcal{X} \to \mathbb{R}_+$ be a mapping such that*

(i) *If for all $x, y \in \mathcal{X}$, $d(x, y) = d(y, x) = 0$, then $x = y$;*
(ii) *For all $x, y \in \mathcal{X}$, $d(x, y) = d(y, x)$;*
(iii) *There exists a real number $b \geq 1$ such that for all $x, y, z \in \mathcal{X}$, $d(x, z) \leq b(d(x, y) + d(y, z))$.*

Then d is called a dislocated b-metric on $\mathcal{X}$ and $(\mathcal{X}, d)$ is called a dislocated b-metric space.

Definition 2 ([10]). *Let $\mathcal{X}$ be a dislocated b-metric space. A sequence $\{x_n\}$ in $\mathcal{X}$ is said to be a Cauchy sequence if $\lim_{m,n\to\infty} d(x_n, x_m) = 0$.*

Definition 3 ([10]). *Let $\mathcal{X}$ be a dislocated b-metric space. A sequence $\{x_n\}$ in $\mathcal{X}$ converges in $\mathcal{X}$ if there exists an element $x \in \mathcal{X}$ such that $\lim_{n\to\infty} d(x_n, x) = 0$. If $\{x_n\}$ converges to x, we write $x_n \to x$ as $n \to \infty$. A dislocated b-metric space in which every Cauchy sequence converges will be called complete.*

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Lemma 1 ([1]). *Let $\{x_n\}$ be a sequence in a complete dislocated b-metric space $\mathcal{X}$. If there exists λ such that $\lambda b \in [0,1)$ and $d(x_n, x_{n+1}) \leq \lambda d(x_{n-1}, x_n)$ for all $n \in \mathbb{N}$, then $\{x_n\}$ converges in $\mathcal{X}$.*

A mapping $S : A \cup B \longrightarrow A \cup B$ is cyclic if $S(A) \subseteq B$ and $S(B) \subseteq A$. For more details on cyclic mappings and fixed point theorems, see [11].

Definition 4 ([12]). *Let $\mathcal{X}$ be a non-empty set and let $d : \mathcal{X} \times \mathcal{X} \to \mathbb{R}_+$ be a mapping such that*

(i) *If for all $x, y \in \mathcal{X}$, $d(x,y) = d(y,x) = 0$, then $x = y$;*
(ii) *There exists a real number $b \geq 1$ such that for all $x, y, z \in \mathcal{X}$, $d(x,z) \leq b(d(x,y) + d(y,z))$.*

Then d is called a dislocated quasi b-metric on $\mathcal{X}$ and $(\mathcal{X}, d)$ is called a dislocated quasi b-metric space.

Definition 5 ([12]). *Let $\mathcal{X}$ be a dislocated quasi b-metric space. A sequence $\{x_n\}$ in $\mathcal{X}$ is said to be a Cauchy sequence if $\lim\limits_{m,n\to\infty} d(x_n, x_m) = \lim\limits_{m,n\to\infty} d(x_m, x_n) = 0$.*

Definition 6 ([12]). *Let $\mathcal{X}$ be a dislocated quasi b-metric space. A sequence $\{x_n\}$ in $\mathcal{X}$ converges in $\mathcal{X}$ if there exists an element $x \in \mathcal{X}$ such that $\lim\limits_{n\to\infty} d(x_n, x) = \lim\limits_{n\to\infty} d(x, x_n) = 0$. If $\{x_n\}$ converges to x, we write $x_n \to x$ as $n \to \infty$. A dislocated quasi b-metric space in which every Cauchy sequence converges will be called complete.*

2. Main Results

Now, we introduce an interpolative Kannan type contraction in a dislocated b-metric space as follows.

Definition 7. *A self-mapping S on a dislocated b-metric space $\mathcal{X}$ is called an interpolative Kannan type contraction if there exist λ with $\lambda b \in [0,1)$ and $\alpha \in (0,1)$ such that*

$$d(Sx, Sy) \leq \lambda [d(x, Sx)]^{\alpha} \cdot [d(y, Sy)]^{1-\alpha} \tag{1}$$

for each $x, y \in \mathcal{X} \backslash Fix(S)$ where $Fix(S) = \{u \in \mathcal{X} : Su = u\}$.

We have the following theorem.

Theorem 1. *Let $\mathcal{X}$ be a complete dislocated b-metric space and let S be an interpolative Kannan type contraction on $\mathcal{X}$. Then S has a fixed point in $\mathcal{X}$.*

Proof. Let $x_0 \in \mathcal{X}$. We construct a sequence $\{x_n\} \subset \mathcal{X}$ by $x_{n+1} = Sx_n$ for each $n \in \mathbb{N}_0$. Without loss of generality, we suppose that $x_{n+1} \neq x_n$ for all $n \in \mathbb{N}_0$. Indeed, if there exist a nonnegative integer n_0 such that $x_{n_0} = x_{n_0+1} = Sx_{n_0}$, then x_{n_0} forms a fixed point. Then

$$d(x_n, x_{n+1}) > 0$$

for all $n \in \mathbb{N}_0$. Putting $x = x_n$ and $y = x_{n-1}$ in (1), we obtain

$$\begin{aligned} d(x_{n+1}, x_n) &= d(Sx_n, Sx_{n-1}) \\ &\leq \lambda [d(x_n, Sx_n)]^{\alpha} \cdot [d(x_{n-1}, Sx_{n-1})]^{1-\alpha} \\ &= \lambda [d(x_n, x_{n+1})]^{\alpha} \cdot [d(x_{n-1}, x_n)]^{1-\alpha}, \end{aligned}$$

then

$$d(x_n, x_{n+1})^{1-\alpha} \leq \lambda d(x_{n-1}, x_n)^{1-\alpha}. \tag{2}$$

Hence

$$d(x_n, x_{n+1}) \leq \lambda^{\frac{1}{1-\alpha}} d(x_{n-1}, x_n) \tag{3}$$

where $\lambda^{\frac{1}{1-\alpha}} < 1$. Since $\lambda b \in [0,1)$, it follows from Lemma 1 that $\{x_n\}$ converges to some $x^* \in \mathcal{X}$. Now, we demonstrate that $x^* \in \mathcal{X}$ is a fixed point of S. Hence

$$d(x_{n+1}, Sx^*) = d(Sx_n, Sx^*) \leq \lambda [d(x_n, x_{n+1})]^{\alpha} \cdot [d(x^*, Sx^*)]^{1-\alpha}.$$

Letting $n \longrightarrow \infty$, we get
$$d(x^*, Sx^*) = 0.$$
Therefore $Sx^* = x^*$ thus x^* is a fixed point of S. □

We obtain the next corollary.

Corollary 1. *Let A and B be two nonempty closed subsets of a complete dislocated b-metric space $\mathcal{X}$. Suppose that $S : A \cup B \longrightarrow A \cup B$ is cyclic such that there exist λ with $\lambda b \in [0, 1)$ and $\alpha \in (0, 1)$ such that*
$$d(Sx, Sy) \leq \lambda[d(x, Sx)]^{\alpha} \cdot [d(y, Sy)]^{1-\alpha} \tag{4}$$
for each $x \in A \backslash Fix(S)$ and $y \in B \backslash Fix(S)$. Then S has a fixed point in $A \cap B$.

Now, we introduce an interpolative Reich-Rus-Ćirić type contraction in complete dislocated b-metric spaces as follows.

Definition 8. *A self-mapping S on a dislocated b-metric space $\mathcal{X}$ is called an interpolative Reich-Rus-Ćirić type contraction if there exist λ with $\lambda b \in [0, 1)$ and $\alpha, \beta \in (0, 1)$ such that for each $x, y \in \mathcal{X} \backslash Fix(S)$,*
$$d(Sx, Sy) \leq \lambda[d(x, y)]^{\beta} \cdot [d(x, Sx)]^{\alpha} \cdot [d(y, Sy)]^{1-\alpha-\beta}. \tag{5}$$

Theorem 2. *Let $\mathcal{X}$ be a complete dislocated b-metric space and let S be an interpolative Reich-Rus-Ćirić type contraction on $\mathcal{X}$. Then S has a fixed point in $\mathcal{X}$.*

Proof. Let $x_0 \in \mathcal{X}$. Define the sequence $\{x_n\} \subset \mathcal{X}$ by $x_{n+1} = Sx_n$ for each $n \in \mathbb{N}_0$. If there exist a nonnegative integer n_0 such that $x_{n_0} = x_{n_0+1} = Sx_{n_0}$, then x_{n_0} forms a fixed point. The proof is complete. Henceforth, suppose that $x_{n+1} \neq x_n$ for all $n \in \mathbb{N}_0$. Setting $x = x_n$ and $y = x_{n-1}$ in (5), we have
$$\begin{aligned} d(x_n, x_{n+1}) &= d(Sx_n, Sx_{n-1}) \\ &\leq \lambda[d(x_n, x_{n-1})]^{\beta} \cdot [d(x_n, Sx_n)]^{\alpha} \cdot [d(x_{n-1}, Sx_{n-1})]^{1-\alpha-\beta} \\ &= \lambda[d(x_n, x_{n-1})]^{\beta} \cdot [d(x_n, x_{n+1})]^{\alpha} \cdot [d(x_{n-1}, x_n)]^{1-\alpha-\beta} \\ &= \lambda[d(x_n, x_{n+1})]^{\alpha} \cdot [d(x_{n-1}, x_n)]^{1-\alpha} \end{aligned} \tag{6}$$
then
$$d(x_n, x_{n+1})^{1-\alpha} \leq \lambda d(x_{n-1}, x_n)^{1-\alpha}. \tag{7}$$
Therefore
$$d(x_n, x_{n+1}) \leq \lambda^{\frac{1}{1-\alpha}} d(x_{n-1}, x_n) \tag{8}$$
where $\lambda^{\frac{1}{1-\alpha}} < 1$. Since $\lambda b \in [0, 1)$, it follows from Lemma 1 that $\{x_n\}$ converges to some $x^* \in \mathcal{X}$. Now, we demonstrate that $x^* \in \mathcal{X}$ is a fixed point of S. Hence
$$\begin{aligned} d(x_{n+1}, Sx^*) &= d(Sx_n, Sx^*) \\ &\leq \lambda[d(x_n, x^*)]^{\beta} \cdot [d(x_n, x_{n+1})]^{\alpha} \cdot [d(x^*, Sx^*)]^{1-\alpha-\beta}. \end{aligned} \tag{9}$$
Letting $n \longrightarrow \infty$ in (9), we obtain
$$d(x^*, Sx^*) = 0.$$
Then $Sx^* = x^*$, thus x^* is a fixed point of S. □

We get the next corollary.

Corollary 2. *Let A and B be two nonempty closed subsets of a complete dislocated b-metric space $\mathcal{X}$. Suppose that $S : A \cup B \longrightarrow A \cup B$ is cyclic such that there exist λ with $\lambda b \in [0, 1)$ and $\alpha, \beta \in (0, 1)$ such that*
$$d(Sx, Sy) \leq \lambda[d(x, y)]^{\beta} \cdot [d(x, Sx)]^{\alpha} \cdot [d(y, Sy)]^{1-\alpha-\beta} \tag{10}$$
for each $x \in A \backslash Fix(S), y \in B \backslash Fix(S)$. Then S has a fixed point in $A \cap B$.

We introduce an interpolative Hardy-Rogers type contraction in complete dislocated b-metric spaces as follows.

Definition 9. *A self-mapping S on a complete dislocated b-metric space $\mathcal{X}$ is called an interpolative Hardy-Rogers type contraction if there exist λ with $\lambda b \in [0,1)$ and $\alpha, \beta, \gamma \in (0,1)$ with $\alpha+\beta+\gamma<1$ such that for each $x, y \in \mathcal{X} \backslash Fix(S)$,*

$$d(Sx, Sy) \leq \lambda[d(x,y)]^{\beta} \cdot [d(x,Sx)]^{\alpha} \cdot [d(y,Sy)]^{\gamma} \cdot [\frac{1}{4b}(d(x,Sy)+d(y,Sx))]^{1-\alpha-\beta-\gamma}. \tag{11}$$

Theorem 3. *Let $\mathcal{X}$ be a complete dislocated b-metric space and let S be an interpolative Hardy-Rogers type contraction on $\mathcal{X}$. Then S has a fixed point in $\mathcal{X}$.*

Proof. Let $x_0 \in \mathcal{X}$. We construct a sequence $\{x_n\} \subset \mathcal{X}$ by $x_{n+1} = Sx_n$ for each $n \in \mathbb{N}_0$. If there exist a nonnegative integer n_0 such that $x_{n_0} = x_{n_0+1} = Sx_{n_0}$, then x_{n_0} forms a fixed point. The proof is complete. Henceforth, suppose that $x_{n+1} \neq x_n$ for all $n \in \mathbb{N}_0$. Putting $x = x_n$ and $y = x_{n-1}$ in (11), we have

$$\begin{aligned} d(x_n, x_{n+1}) &= d(Sx_n, Sx_{n-1}) \\ &\leq \lambda[d(x_n, x_{n-1})]^{\beta} \cdot [d(x_n, Sx_n)]^{\alpha} \cdot [d(x_{n-1}, Sx_{n-1})]^{\gamma} \\ &\quad \cdot [\frac{1}{4b}(d(x_n, x_n) + d(x_{n-1}, x_{n+1}))]^{1-\alpha-\beta-\gamma} \\ &= \lambda[d(x_n, x_{n-1})]^{\beta} \cdot [d(x_n, x_{n+1})]^{\alpha} \cdot [d(x_{n-1}, x_n)]^{\gamma} \\ &\quad \cdot [\frac{1}{4b}(2bd(x_{n-1}, x_n) + d(x_{n-1}, x_{n+1}))]^{1-\alpha-\beta-\gamma}. \end{aligned} \tag{12}$$

Suppose that $d(x_{n-1}, x_n) \leq d(x_n, x_{n+1})$. Then from (12), we obtain

$$d(x_n, x_{n+1})^{\beta+\gamma} \leq \lambda d(x_{n-1}, x_n)^{\beta+\gamma}$$

which is a contradiction. Then $\{d(x_n, x_{n+1})\}$ is decreasing. From (12),

$$d(x_n, x_{n+1}) \leq \lambda d(x_{n-1}, x_n) \tag{13}$$

where $\lambda < 1$. Since $\lambda b \in [0,1)$, it follows from Lemma 1 that $\{x_n\}$ converges to some $x^* \in \mathcal{X}$. Now, we demonstrate that $x^* \in \mathcal{X}$ is a fixed point of S. Hence

$$\begin{aligned} d(x_{n+1}, Sx^*) &= d(Sx_n, Sx^*) \\ &\leq \lambda[d(x_n, x^*)]^{\beta} \cdot [d(x_n, Sx_n)]^{\alpha} \cdot [d(x^*, Sx^*)]^{\gamma} \\ &\quad \cdot [\frac{1}{4b}(d(x_n, Sx^*) + d(x^*, Sx_n))]^{1-\alpha-\beta-\gamma}. \end{aligned} \tag{14}$$

Letting $n \longrightarrow \infty$ in (14), we obtain

$$d(x^*, Sx^*) = 0.$$

Then $Sx^* = x^*$ thus x^* is a fixed point of S. □

Corollary 3. *Let A and B be two nonempty closed subsets of a complete dislocated b-metric space $\mathcal{X}$. Suppose that $S : A \cup B \longrightarrow A \cup B$ is cyclic such that there exist λ with $\lambda b \in [0,1)$ and $\alpha, \beta, \gamma \in (0,1)$ with $\alpha+\beta+\gamma<1$ such that*

$$\begin{aligned} d(Sx, Sy) &\leq \lambda[d(x,y)]^{\beta} \cdot [d(x,Sx)]^{\alpha} \cdot [d(y,Sy)]^{\gamma} \\ &\quad \cdot [\frac{1}{4b}(d(x,Sy)+d(y,Sx))]^{1-\alpha-\beta-\gamma} \end{aligned} \tag{15}$$

for each $x \in A \backslash Fix(S), y \in B \backslash Fix(S)$. Then S has a fixed point in $A \cap B$.

Now, we introduce an interpolative Kannan-Ćirić type contraction in a complete dislocated b-metric space as follows.

Definition 10. *A mapping S on a dislocated b-metric space $\mathcal{X}$ is called an interpolative Kannan-Ćirić type contraction if there exists $\alpha \in (0,1)$ such that for each $x, y \in \mathcal{X} \backslash Fix(S)$, we have*

(i) *given $\varepsilon > 0$, there exists $\delta > 0$ such that*

$$\varepsilon < [d(x, Sx)]^{\alpha}[d(y, Sy)]^{1-\alpha} < \varepsilon + \delta \Rightarrow d(Sx, Sy) \leq \varepsilon \tag{16}$$

and

(ii)

$$d(Sx, Sy) < [d(x, Sx)]^{\alpha}[d(y, Sy)]^{1-\alpha}. \tag{17}$$

Now, we present the next result.

Theorem 4. *Let $\mathcal{X}$ be a complete dislocated b-metric space and let S be an interpolative Kannan-Ćirić type contraction on $\mathcal{X}$. Then S has a fixed point in $\mathcal{X}$.*

Proof. Let $x_0 \in \mathcal{X}$. Define a sequence $\{x_n\} \subset \mathcal{X}$ by $x_{n+1} = Sx_n$ for each $n \in \mathbb{N}_0$. Putting $x = x_{n-1}$ and $y = x_n$ in (17), we have

$$\begin{aligned} d(x_n, x_{n+1}) &= d(Sx_{n-1}, Sx_n) \\ &< [d(x_{n-1}, Sx_{n-1})]^{\alpha}[d(x_n, Sx_n)]^{1-\alpha} \\ &= [d(x_{n-1}, x_n)]^{\alpha}[d(x_n, x_{n+1})]^{1-\alpha}. \end{aligned} \tag{18}$$

Then $[d(x_n, x_{n+1})]^{\alpha} < [d(x_{n-1}, x_n)]^{\alpha}$, thus $\{d(x_n, x_{n+1})\}$ is strictly decreasing. As a result, there exists $M \in \mathbb{R}_+$ such that $\lim\limits_{n\to\infty} d(x_n, x_{n+1}) = M$. We claim that $M = 0$. By monotonicity, we have

$$d(x_n, x_{n+1}) > M \quad \text{for all } n \in \mathbb{N}. \tag{19}$$

Indeed, if we suppose that $M > 0$, we can find $N \in \mathbb{N}$ such that

$$M < d(x_n, x_{n+1}) < M + \delta(M) \tag{20}$$

for all $n \geq N$. Since $M < d(x_n, x_{n+1}) < [d(x_{n-1}, x_n)]^{\alpha}[d(x_n, x_{n+1})]^{1-\alpha}$, it follows from (16) that $d(x_n, x_{n+1}) \leq M$ for any $n \geq N$ which is a contradiction with (19). So $M = 0$. Now, we demonstrate that $\{x_n\}$ is Cauchy. Let $\varepsilon > 0$. Since $\lim\limits_{n\longrightarrow\infty} d(x_n, x_{n+1}) = 0$, we can find $N_1 \in \mathbb{N}$ such that $d(x_n, x_{n+1}) < \frac{\varepsilon}{2b}$ for $n \geq N_1$. Then

$$\begin{aligned} d(x_n, x_{n+p}) &\leq b(d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+p})) \\ &= b(d(x_n, x_{n+1}) + d(Sx_n, Sx_{n+p-1})) \\ &< b(d(x_n, x_{n+1}) + [d(x_n, x_{n+1})]^{\alpha}[d(x_{n+p-1}, x_{n+p})]^{1-\alpha}) \\ &< b(d(x_n, x_{n+1}) + [d(x_n, x_{n+1})]^{\alpha}[d(x_n, x_{n+1})]^{1-\alpha}) \\ &< b(d(x_n, x_{n+1}) + d(x_n, x_{n+1})) \\ &< b(\frac{\varepsilon}{2b} + \frac{\varepsilon}{2b}) \\ &= \varepsilon. \end{aligned}$$

Hence $\{x_n\}$ is Cauchy. Then $\{x_n\}$ converges to some $x^* \in \mathcal{X}$. Now, we demonstrate that $x^* \in \mathcal{X}$ is a fixed point of S. Hence

$$\begin{aligned} d(x_{n+1}, Sx^*) &= d(Sx_n, Sx^*) \\ &< [d(x_n, Sx_n)]^{\alpha}[d(x^*, Sx^*)]^{1-\alpha} \\ &= [d(x_n, x_{n+1})]^{\alpha}[d(x^*, Sx^*)]^{1-\alpha}. \end{aligned} \tag{21}$$

Letting $n \longrightarrow \infty$ in (21), we obtain

$$d(x^*, Sx^*) = 0.$$

Then $Sx^* = x^*$ thus x^* is a fixed point of S. □

We introduce an interpolative Kannan-Meir-Keeler type contraction in dislocated b-metric spaces as follows.

Definition 11. *A self-mapping S on a dislocated b-metric space $\mathcal{X}$ is called an interpolative Kannan-Meir-Keeler type contraction if there exists $\alpha \in (0,1)$ such that for each $x, y \in \mathcal{X} \backslash Fix(S)$, we have*

(i) *given $\varepsilon > 0$, there exists $\delta > 0$ such that*

$$\varepsilon \leq [d(x, Sx)]^{\alpha}[d(y, Sy)]^{1-\alpha} < \varepsilon + \delta \Rightarrow d(Sx, Sy) < \varepsilon, \tag{22}$$

and

(ii)

$$d(Sx, Sy) < [d(x, Sx)]^{\alpha}[d(y, Sy)]^{1-\alpha}. \tag{23}$$

We get the following theorem.

Theorem 5. *Let $\mathcal{X}$ be a complete dislocated b-metric space and let S be an interpolative Kannan-Meir-Keeler type contraction on $\mathcal{X}$. Then S has a fixed point in $\mathcal{X}$.*

Proof. The proof is similar to the proof of Theorem 4. □

We generalize Definition 7 as follows.

Definition 12. *A continuous self-mapping S on a dislocated quasi b-metric space $\mathcal{X}$ is called an interpolative Kannan type contraction if there exist λ with $\lambda b \in [0,1)$ and $\alpha \in (0,1)$ such that*

$$d(Sx, Sy) \leq \lambda[d(x, Sx)]^{\alpha} \cdot [d(y, Sy)]^{1-\alpha}$$

for each $x, y \in \mathcal{X} \backslash Fix(S)$ where $Fix(S) = \{u \in \mathcal{X} : Su = u\}$.

As a generalization of Theorem 1, we obtain:

Theorem 6. *Let $\mathcal{X}$ be a complete dislocated quasi b-metric space and let S be an interpolative Kannan type contraction on $\mathcal{X}$. Then S has a fixed point in $\mathcal{X}$.*

We obtain the next corollary.

Corollary 4. *Let A and B be two nonempty closed subsets of a complete dislocated quasi b-metric space $\mathcal{X}$. Suppose that $S : A \cup B \longrightarrow A \cup B$ is cyclic such that there exist λ such that $\lambda b \in [0,1)$ and $\alpha \in (0,1)$ with*

$$d(Sx, Sy) \leq \lambda[d(x, Sx)]^{\alpha} \cdot [d(y, Sy)]^{1-\alpha} \tag{24}$$

for each $x \in A \backslash Fix(S)$ and $y \in B \backslash Fix(S)$. Then S has a fixed point in $A \cap B$.

We generalize Definition 8 as follows.

Definition 13. *A continuous self-mapping S on a dislocated quasi b-metric space $\mathcal{X}$ is called an interpolative Reich-Rus-Ćirić type contraction if there exist λ satisfying $\lambda b \in [0,1)$ and $\alpha, \beta \in (0,1)$ such that for each $x, y \in \mathcal{X} \backslash Fix(S)$,*

$$d(Sx, Sy) \leq \lambda[d(x, y)]^{\beta} \cdot [d(x, Sx)]^{\alpha} \cdot [d(y, Sy)]^{1-\alpha-\beta}.$$

The following theorem generalizes Theorem 2.

Theorem 7. *Let $\mathcal{X}$ be a complete dislocated quasi b-metric space and let S be an interpolative Reich-Rus-Ćirić type contraction on $\mathcal{X}$. Then S has a fixed point in $\mathcal{X}$.*

Corollary 5. *Let A and B be two nonempty closed subsets of a complete dislocated quasi b-metric space $\mathcal{X}$. Suppose that $S : A \cup B \longrightarrow A \cup B$ is cyclic such that there exist λ such that $\lambda b \in [0, 1)$ and $\alpha, \beta \in (0, 1)$ such that*

$$d(Sx, Sy) \leq \lambda[d(x, y)]^{\beta} \cdot [d(x, Sx)]^{\alpha} \cdot [d(y, Sy)]^{1-\alpha-\beta} \tag{25}$$

for each $x \in A \backslash Fix(S), y \in B \backslash Fix(S)$. Then S has a fixed point in $A \cap B$.

We generalize Definition 9 as follows.

Definition 14. *A continuous self-mapping S on a complete dislocated quasi b-metric space $\mathcal{X}$ is called an interpolative Hardy-Rogers type contraction if there exist λ with $\lambda b \in [0, 1)$ and $\alpha, \beta, \gamma \in (0, 1)$ with $\alpha + \beta + \gamma < 1$ such that for each $x, y \in \mathcal{X} \backslash Fix(S)$,*

$$\begin{aligned} d(Sx, Sy) \leq \lambda[d(x, y)]^{\beta} \cdot [d(x, Sx)]^{\alpha} \cdot [d(y, Sy)]^{\gamma} \\ \cdot [\frac{1}{4b}(d(x, Sy) + d(y, Sx))]^{1-\alpha-\beta-\gamma}. \end{aligned}$$

The next theorem generalizes Theorem 3.

Theorem 8. *Let $\mathcal{X}$ be a complete dislocated quasi b-metric space and let S be an interpolative Hardy-Rogers type contraction on $\mathcal{X}$ which is asymptotically regular. Then S has a fixed point in $\mathcal{X}$.*

Corollary 6. *Let A and B be two nonempty closed subsets of a complete dislocated quasi b-metric space $\mathcal{X}$. Suppose that $S : A \cup B \longrightarrow A \cup B$ is cyclic and asymptotically regular such that there exist λ with $\lambda b \in [0, 1)$ and $\alpha, \beta, \gamma \in (0, 1)$ with $\alpha + \beta + \gamma < 1$ such that*

$$d(Sx, Sy) \leq \lambda[d(x, y)]^{\beta} \cdot [d(x, Sx)]^{\alpha} \cdot [d(y, Sy)]^{\gamma} \cdot [\frac{1}{4b}(d(x, Sy) + d(y, Sx))]^{1-\alpha-\beta-\gamma} \tag{26}$$

for each $x \in A \backslash Fix(S), y \in B \backslash Fix(S)$. Then S has a fixed point in $A \cap B$.

We generalize Definition 10 as follows.

Definition 15. *A continuous self-mapping S on a dislocated quasi b-metric space $\mathcal{X}$ is called an interpolative Kannan-Ćirić type contraction if there exists $\alpha \in (0, 1)$ such that for each $x, y \in \mathcal{X} \backslash Fix(S)$, we have*

(i) *given $\varepsilon > 0$, there exists $\delta > 0$ such that*

$$\varepsilon < [d(x, Sx)]^{\alpha}[d(y, Sy)]^{1-\alpha} < \varepsilon + \delta \Rightarrow d(Sx, Sy) \leq \varepsilon$$

and

(ii)

$$d(Sx, Sy) < [d(x, Sx)]^{\alpha}[d(y, Sy)]^{1-\alpha}.$$

The next theorem generalizes Theorem 4.

Theorem 9. *Let $\mathcal{X}$ be a complete dislocated quasi b-metric space and let S be an interpolative Kannan-Ćirić type contraction on $\mathcal{X}$. Then S has a fixed point in $\mathcal{X}$.*

We generalize Definition 11 as follows.

Definition 16. *A continuous self-mapping S on a dislocated quasi b-metric space $\mathcal{X}$ is called an interpolative Kannan-Meir-Keeler type contraction if there exists $\alpha \in (0, 1)$ such that for each $x, y \in \mathcal{X} \backslash Fix(S)$, we have*

(i) *given $\varepsilon > 0$, there exists $\delta > 0$ such that*

$$\varepsilon \leq [d(x, Sx)]^{\alpha}[d(y, Sy)]^{1-\alpha} < \varepsilon + \delta \Rightarrow d(Sx, Sy) < \varepsilon, \tag{27}$$

and

(ii)
$$d(Sx, Sy) < [d(x, Sx)]^{\alpha}[d(y, Sy)]^{1-\alpha}. \tag{28}$$

We finish with the following theorem as a generalization of Theorem 5.

Theorem 10. *Let $\mathcal{X}$ be a complete dislocated quasi b-metric space and let S be an interpolative Kannan-Meir-Keeler type contraction on $\mathcal{X}$. Then S has a fixed point in $\mathcal{X}$.*

Statements and Declarations

The author declares that he has no conflict of interest, and the manuscript has no associated data.

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